

## Problem Sheet 1.

There is an island in the middle of nowhere inhabited only by Liars and Knights. Liars always lie and Knights always tell the truth. You were left on that island (as a consequence of terrible circumstances), and now you have to survive.

It is vital for your survival to know how to distinguish between Liars and Knights. You cannot judge from their appearance. Fortunately for you, you can speak their language. You can ask them questions, and the native inhabitants of the island are friendly enough to answer.

Here are some examples.

**Example 1.** Could you formulate a question which both a liar and a knight would answer identically (either "Yes" or "No")?

**Example 2.** Inhabitant A tells Inhabitant B, "At least one of us is a liar". Who is A and who is B?

**Example 3.** Three inhabitants are passing by. You ask the three inhabitants, "How many among you are knights?"

The first one replies, "There is none".

The second inhabitant argues, "There is only one".

What should the third inhabitant say?

**Problem 1.1.** John is a knight, he never lies. But when you ask him the same question twice, his second answer suddenly is different from the first. How is it possible?

**Problem 1.2.** a) Twelve inhabitants are sitting under a big tree. One of them says, "There is at least one liar among us". How many knights can be among them?

b) A different group of other twelve inhabitants is resting by the river. Suddenly, one of them exclaims, "If everyone except me is a liar, then I am a liar too". How many liars can be among them?

**Problem 1.3.** Ten inhabitants are sitting around the table. Each one of them says, "My neighbour on the right is a knight". How many knights and liars are sitting there?

**Problem 1.4.** Albert (A), his wife Beatrix (B), and their children Charlie (C), Dan (D) and Elizabeth (E) live in a bungalow. They have a really nice TV set. It is known that

- 1) If A is watching the TV, then B is watching the TV.
- 2) At least one of D and E is watching the TV.
- 3) Only one of B and C is watching the TV.
- 4) Either C and D are watching the TV together, or both are not watching.
- 5) If E is watching the TV, then both A and D are also watching the TV.

Can you tell who is watching the TV in this family and who is not?

**Problem 1.5.** Once I found a really strange notebook. There were 100 statements in the notebook, namely

"There is exactly one false statement in this notebook."

"There are exactly two false statements in this notebook."

"There are exactly three false statements in this notebook."

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"There are exactly one hundred false statements in this notebooks."

Are there any true statements in this notebook? If there are some true statements, then which ones are true?