Problem Sheet 6.

Definition. A *divisibility rule* is a shorthand way of determining whether a given number is divisible by a fixed divisor without performing the division. This is usually done by examining its digits.

Divisibility rules:
for 2: a number is divisible by 2 if the last digit is even (0, 2, 4, 6, 8);
for 3: a number is divisible by 3 if the sum of the digits is divisible by 3;
for 5: a number is divisible by 5 if the last digit is 5 or 0;
for 9: a number is divisible by 9 if the sum of the digits is divisible by 9;
for 4: a number is divisible by 4 if the number made by the last two digits is divisible by 4;
for 25: a number is divisible by 25 if the number made by the last two digits is divisible by 25.

Moreover, if a number is divisible by 2, then the last digit is even. Similar statements are true for the other divisibility rules.

Example 1. Prove the divisibility rule for 3.

Example 2. Is it possible for n! to be written as 2015000...000, where the number of 0's at the end can be arbitrary?

Problem 6.1. Was it clear about the divisibility rule for 3? Now prove the divisibility rule for 9 by using the same arguments.

Problem 6.2. A battle of the captains was held at a maths battle. The task was to write the smallest number such that it is divisible by 45 and consists of only 1's and 0's as digits. What do you think was the correct answer?

Problem 6.3. It is known that a natural number is three times bigger than the sum of its digits. Is it divisible by 27?

Problem 6.4. A number was left written on the white board after a maths class. The number consisted of one hundred 0's, one hundred 1's, and one hundred 2's as digits. A cleaner was about to wipe it off when suddenly he saw a small comment written in a corner. The comment stated that the number was a square number. He fetched a sigh and wrote "it not a square number". Why was he right?

Problem 6.5. A stoneboard was found on the territory of the ancient Greek Academia as a result of archaeological excavations. The archeologists decided that this stoneboard belonged to a mathematician who lived in the 7th century BC. The list of unsolved problems was written on the stoneboard. The archaeologists became thrilled to solve the problems but got stuck on the fifth. They were looking for a 10-digit number. The number should consist of only different digits. Moreover, if you cross any 6 digits, the remaining number should be composite. Can you help the archeologists to figure out the answer?