

Additional Problems.

Problem 8.7. Graphical application works in the following way. At the beginning user draws a point. In one second the program draws 3 vertices of an equilateral triangle with a centre which coincides with the initial point and deletes this point. In one second the program does the procedure described above for each of the three new points and deletes them after. So we get 9 points instead. This application runs until we stop it. Every time all new points are different from each other and from previous points. Will there be a moment for us to stop the program so that the current number of points is not equal to 9 and is 9 modulo 97?

Problem 8.8. Before leaving the Middle-Earth, Frodo handed his Hobbit-Hole down to Sam in order for his friend to settle down for life. The apartment consisted of 11 rooms. The areas of all rooms (if counted in square meters) were different natural numbers, each of which was not greater than 20. Prove that there were two rooms with areas a and b , such that a divides b .

Problem 8.9. You are given 10 integers. Prove that you can always choose such numbers among them, that the sum of the chosen numbers is divisible by 10.

Problem 8.10. In a herd of 101 cows, each cow weighs an integer number of kilograms. If a cow is removed from the herd, the remaining cows can be divided into two groups of 50 cows each, with the total weight of all the cows in both groups being equal. Prove that all the cows weigh the same.

Problem 8.11. Each of the letters F, I, V, E in this multiplication stands for a different digit:

$$\begin{array}{rcccccc}
 & & & & F & I & V & E \\
 & & & & \times & F & I & V & E \\
 & & & & \hline
 & & & * & * & * & * & F \\
 & & * & * & * & * & I \\
 & * & * & * & * & V \\
 * & * & * & * & E \\
 \hline
 * & * & * & * & * & * & * & *
 \end{array}$$

What are the values of the letters?