# Maths Battle 2

The Royal Grammar School, High Wycombe

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Problem 1. Neptune, the king of sea, has various octopuses as his servants. Some octopuses have 8 legs, some have 7 legs, and some have only 6 legs. Seven-legged octopuses always lie, but octopuses with 6 or 8 legs always tell the truth. Once upon a time 4 octopuses bumped into each other while doing some errands for Neptune. The blue octopus said, "We have 28 legs between us". The green octopus argued,"We have 27 legs between us". The yellow octopus did not agree, "We have 26 legs between us". The red octopus objected, "We have 25 legs between us". How many legs does each of them have?

Problem 2. Is it possible to construct a rectangle with edges longer than 1 using all the 13 rectangles 1×1, 1×2, ..., 1×13?

Problem 3. In the centre of Whoville, there exists a Christmas cube $3 \times 3 \times 3$. Grinch by accident broke this magnificent cube into several pieces: 7 corners consisting of 3 cubes $1 \times 1 \times 1$ and 6 separate cubes $1 \times 1 \times 1$.
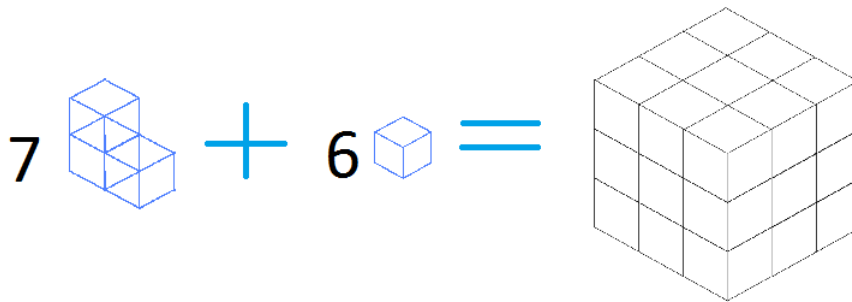
How to reconstruct the initial cube from these broken pieces to save Christmas?

Problem 4. There are 8 numbers written in a circle. Between each pair of neighbouring numbers Rudolph writes their sum and erases the initial numbers. After his manipulations only the numbers 11, 12, 13, 14, 15, 16, 17, 18 are left written in a circle in this particular order. Is it possible?

Problem 5. On the night of Christmas Eve, a little girl goes up to the wall to rip the page of the calender. She is very excited that it will be Christmas tomorrow and rips the page halfway and stops to ponder (see the picture).
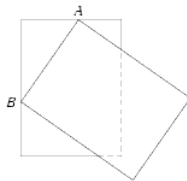
The vertices A and B lay on the sides of the bottom sheet as it is shown in the picture. The fourth vertex of the bottom sheet cannot be seen as it is covered by the top sheet. The top and the bottom sheets are completely identical. Which part of the bottom sheet is bigger - covered or uncovered?

Problem 6. One day, a young mathematician wrote a letter to Santa. In that letter he asked Santa if it was possible to put one of the numbers 0,-1,1 in each box of the table 6×6 in such a way that every row, every column and both main diagonals have different sum of numbers of their entries. What was Santa's answer? (As you know Santa is very bright and never lies to children).

Problem 7. Twenty-five elves and 25 dwarves are seated around a table. Show that at least one creature has elves as both neighbours.

Problem 8. A five-digit number is called "sustainable" if it cannot be written as a product of two 3-digit numbers. How many consecutive "sustainable" numbers do exist?