Maths Battle 3

The Royal Grammar School, High Wycombe

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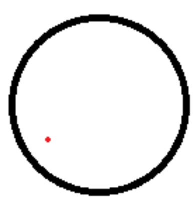
Problem 1. Once Michael was applying for a teaching position in his old school. During the interview, he was asked to compare two numbers and to find the difference between them. Michael was not very good at calculations at that time, but somehow he managed to give the correct answer. The numbers were $333333333333 \times 888888888888$ and $444444444444 \times 666666666667$. How did he do it?

Problem 2. In Gerrard's box there are 100 stars which are red, green or yellow. Three of these stars are magic, they can change their colour at any time. Once Gerrard took a look at his box and realised that the number of red stars was greater than the number of yellow stars, and the number of yellow stars was greater than the number of green stars. The second time he opened his box the situation was the opposite: the number of green stars was greater than the number of yellow stars, and the number of yellow stars was greater than the number of red stars. Can you determine how many yellow stars were in the box when Gerrard opened it for the first time?

Problem 3. A warehouse contains 200 single boots of size 8, 200 single boots of size 9, and 200 single boots of size 10, all together 600 individual boots. There are 300 left boots and 300 right boots among them. Show that it is possible to find at least 100 matching pairs among these boots if all the boots are of the same style and colour.

Problem 4. A dot is marked inside a circle. Cut the circle into two parts so that you can construct a circle with its center at the dot by rearranging the parts.



Problem 5. Cards numbered from 1 to 1000 are arranged face up on a table. Alex and Bob are taking these cards one by one until there are none left. Alex is making the first move and his goal is to ensure that at the end of the game the sum of the numbers on his cards is a multiple of 25. Can Bob ruin his plans?

Problem 6. It is written in The Guinness Book of Records that there exist three natural numbers such that each of them is not divisible by the others, but the square of every number is divisible by any other number. Improve this result by suggesting 10 natural numbers satisfying the same conditions.